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A new overtaking model and numerical tests

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Abstract

A new model for overtaking on highway is proposed. The model considers such important factors as the reactive delay time for vehicle acceleration, deceleration, and lane-changing, the safe distance for car-following and the distance for overtaking. The time required for overtaking, the time loss in overtaking procedure and the space–time evolution of vehicle movement are numerically investigated using the model and compared with the results from a survey. Numerical results show that our model can generate the traffic in accord with the observed one. The overtaking in a two-lane bidirectional traffic flow is also analyzed.

Keywords: Overtaking; Safe distance; Reactive delay time; Overtaking distance

1. Introduction

Overtaking is a common phenomenon on highway. The main approaches that are used to study overtaking are cellular automata modeling [1–6] and differential equation modeling [7–9]. In addition, the system theoretic approach and the neural network method are applied to study the human operating behavior in overtaking procedure [10–12]. Although these models can explain some overtaking behavior, they are unable to formulate an analytical expression of the fast vehicle's velocity changes in overtaking. Recently, the overtaking distance-based approach has attracted attention [13–16], but it is not practicable due to the difficulty of calibrating too many parameters.

For formulating the overtaking on highway and analytically deriving the fast vehicle's velocity changes in overtaking, Xue and Gu [17,18] recently proposed a fixed-end beam deflection curve model. However, this model has such a problem that the larger the difference between the velocities of fast vehicle A and slow vehicle B, the greater the time $\Delta t^{(A)}$ that the fast vehicle A loses in overtaking. In addition, the model assumes that the time T required for overtaking is a constant. Obviously, either the assumption or the result associated

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with this model is not accordant with the reality. The observation shows that $\Delta t^{(A)}$ and T should decrease with the difference $v^{(A)} - v^{(B)}$, where $v^{(A)}$ and $v^{(B)}$ are the velocities of vehicles A and B, respectively.

This paper proposes a new overtaking model in which the reactive delay time for vehicle acceleration, deceleration and lane-changing [19,20], the safe distance for car-following, the overtaking distance and the travel distance of the slow vehicle are taken into account. Using the new model, we numerically investigate the time required for overtaking, the time loss in overtaking, the space–time evolution of vehicle movement, and the overtaking distance. We explicitly demonstrate the order of overtaking when several overtaking maneuvers occur. Through comparing our results with those given by the Xue–Gu model, we show that our model can better reproduce the overtaking behavior. We also study the overtaking in a two-lane bidirectional traffic flow and obtain the minimal passing sight distance for overtaking the preceding vehicle by using the opposite lane.

2. The model

The basic overtaking procedure can be described as follows. A fast vehicle A with a speed $v_{\max}^{(A)}$ gains on a slow vehicle B with a speed $v^{(B)}(v_{\max}^{(A)} > v^{(B)})$. When the distance between them decreases to a safe length, vehicle A slows down (although its speed is still greater than $v^{(B)}$) and moves to the side of vehicle B. Vehicle A then accelerates, overtakes vehicle B, and finally regains its original speed $v_{\max}^{(A)}$. The shape of the speed–time curve of vehicle A is a fixed-end beam deflection curve, which was used in Xue and Gu [17,18] for describing the speed of vehicle A in overtaking, i.e.,

$$\Delta v^{(A)}(t) = \begin{cases} -(v_{\max}^{(A)} - v_{\min}^{(A)})\left(3\left(\dfrac{t}{\alpha T}\right)^2 - 2\left(\dfrac{t}{\alpha T}\right)^3\right), & 0 \leqslant t < \alpha T, \\ -(v_{\max}^{(A)} - v_{\min}^{(A)})\left(3\left(\dfrac{T-t}{(1-\alpha)T}\right)^2 - 2\left(\dfrac{T-t}{(1-\alpha T)}\right)^3\right), & \alpha T \leqslant t \leqslant T, \end{cases} \tag{1}$$

where $\Delta v^{(A)}(t) = v^{(A)}(t) - v_{\max}^{(A)}$ represents the change in speed of vehicle A against its original speed $v_{\max}^{(A)}$, and $v_{\min}^{(A)}$ is the lowest speed at which vehicle A can overtake vehicle B (equal to $v^{(B)}$). Clearly, $\Delta v^{(A)}(t) \leqslant 0$ holds in overtaking duration. In Eq. (1), T is the time required by vehicle A for completing the overtaking maneuver from deceleration to recovery of the original speed. Vehicle A decelerates in period $[0, \alpha T]$ and accelerates in period $[\alpha T, T]$, where $0 < \alpha < 1$. By integrating Eq. (1), we can obtain the distance loss of vehicle A because of overtaking, i.e.,

$$\Delta s^{(A)} = \int_0^T \Delta v^{(A)}(t)\,\mathrm{d}t = \int_0^T v^{(A)}(t)\,\mathrm{d}t - \int_0^T v_{\max}^{(A)}\,\mathrm{d}t = 0.5 v_{\max}^{(A)}(1 - v_{\min}^{(A)}/v_{\max}^{(A)})T, \tag{2}$$

where $\Delta s^{(A)}$ is the total distance loss caused by overtaking in comparison with the ideal condition of traveling unimpeded at the maximum speed. Letting $\Delta t^{(A)}$ denote the time loss of vehicle A due to overtaking, we then have

$$\Delta s^{(A)} = v_{\max}^{(A)} \Delta t^{(A)}. \tag{3}$$

Combining Eqs. (2) and (3), we get

$$T = 2\Delta t^{(A)}/(1 - v_{\min}^{(A)}/v_{\max}^{(A)}). \tag{4}$$

Therefore,

$$\Delta t^{(A)} = 0.5T(1 - v_{\min}^{(A)}/v_{\max}^{(A)}) = 0.5T(1 - v^{(B)}/v_{\max}^{(A)}). \tag{5}$$

Eq. (5) shows that $\Delta t^{(A)}$ is a decreasing function of $v^{(B)}$, or that the larger the value of $v^{(B)}$, the smaller the value of $\Delta t^{(A)}$. This is clearly unrealistic. The reason is the assumption that time T is a constant, but observation shows that the time T required for completing an overtaking maneuver should be an increasing function of $v^{(B)}$. Furthermore, other factors, including the overtaking distance, the safe distance for car-following, the vehicle's reactive delay time for deceleration, acceleration and lane-changing, and the distance from the slow vehicle, should be considered in the model. Our new model will consider these factors.

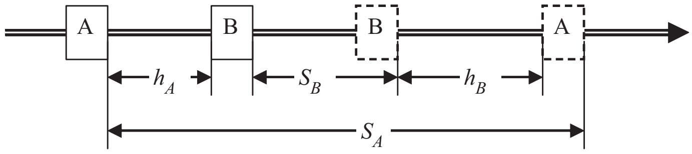

Fig. 1. Positions of the two vehicles before and after overtaking.

Fig. 1 shows the positions of vehicles A and B before and after overtaking. From Eq. (1), we know that the average speed of vehicle A in overtaking is $0.5(v_{\max}^{(A)} + v_{\min}^{(A)})$. Xue and Gu [17,18] simply set the distance that vehicle A travels during time $[0,T]$ be $0.5(v_{\max}^{(A)} + v_{\min}^{(A)})T$, where the reactive delay time is neglected. In fact, the real travel time of the vehicle is $T - 2t_0$, where t_0 is the reactive delay time for deceleration, acceleration and lane-changing. In this paper, we assume $t_0 = 3\,\mathrm{s}$ [19,20].

Note that $2t_0$ is much less than T, the average speed of vehicle A in overtaking can be approximately equal to $0.5(v_{\max}^{(A)} + v_{\min}^{(A)})$. Then, the real distance (the overtaking distance) that vehicle A travels in overtaking is

$$S_A = \frac{v_{\max}^{(A)} + v_{\min}^{(A)}}{2}(T - 2t_0). \tag{6}$$

Suppose that the speed of vehicle B remains unchanged in the overtaking duration, then the distance that vehicle B travels in overtaking is

$$S_B = v^{(B)}T. \tag{7}$$

Neglecting the vehicle's physical length, from Fig. 1, we have the following relationship between the travel distances of the two vehicles,

$$\frac{v_{\max}^{(A)} + v_{\min}^{(A)}}{2}(T - 2t_0) = v^{(B)}T + h_A + h_B, \tag{8}$$

which, with $v_{\min}^{(A)} = v^{(B)}$, leads to

$$T = \frac{2h_A + 2h_B + 2t_0(v_{\max}^{(A)} + v^{(B)})}{v_{\max}^{(A)} - v^{(B)}}. \tag{9}$$

Combining this with Eq. (5) gives

$$\Delta t^{(A)} = \frac{h_A + h_B + t_0(v_{\max}^{(A)} + v^{(B)})}{v_{\max}^{(A)}}, \tag{10}$$

and the overtaking distance of vehicle A can thus be written as follows:

$$S_A = h_A + h_B + \frac{2h_A + 2h_B + 2t_0(v_{\max}^{(A)} + v^{(B)})}{v_{\max}^{(A)} - v^{(B)}}v^{(B)}, \tag{11}$$

where h_A and h_B are the safe distances for car-following of vehicles A and B, respectively. In general, $h_A = 2v_{\max}^{(A)}$ and $h_B = 2v_{\max}^{(B)}$ can be taken, i.e., a two-second gap between vehicles is assumed [19,20].

The overtaking model governed by Eqs. (9)–(11) can reflect the traffic movement more realistically than the original Xue and Gu model. The time $\Delta t^{(A)}$, the time T and the overtaking distance S_A are all increasing functions of the slow vehicle's speed.

3. Numerical tests

Suppose that a car travels along a 15-km highway at a speed of 120 km/h, and over this distance successively overtakes six trucks that are traveling at speeds of 75, 81, 85, 90, 92, and 96 km/h. Assume that none of the trucks changes speed, and that the distance between any two successive trucks is long enough to allow the car to accelerate to $v_{\max}^{(A)}$ before the next overtaking maneuver.

We can, respectively, use Eqs. (9)–(11) to compute T, $\Delta t^{(A)}$, and S_A for each overtaking maneuver. The results are given in Table 1 and depicted in Figs. 2–4, along with the results of the Xue–Gu model for comparison. The time required to traverse the highway without overtaking is $3600(15/120) = 450$ s. However, when overtaking is taken into account, this time given by our model is 501.625 s. The observed time in reality is about 502 s [17]. Table 1 and Figs. 2–4 clearly show that the deficiencies existing in the original Xue–Gu model are removed. These are summarized as follows. (i) The values of $\Delta t^{(A)}$ and T given by our model increase with the speeds of slow vehicles, whereas the value of $\Delta t^{(A)}$ by the Xue–Gu model decreases notably with the speeds of slow vehicles and T is constant. (ii) The overtaking distance S_A given by both models increases with the speeds of slow vehicles, but that by our model increases nonlinearly and remarkably whereas that by the Xue–Gu model increases slightly. Obviously, the new model correctly describes the traffic movement and the overtaking behavior.

We further analyze the space–time evolution of each vehicle. As long as the initial position is given, the analytical formula of each vehicle's speed can be obtained from the new model and the space–time position of each vehicle can then be determined. For demonstration, we here discuss the space–time evolution of vehicles in a uniform flow, and thus assume that the initial headway between two successive vehicles is 250 m and all trucks are distributed along the highway in an ascending order of speeds (that is, the initial positions of all trucks with speeds 75, 81, 85, 90, 92, and 96 km/h are 250, 500, 750, 1000, 1250, and 1500 m from the highway origin, respectively). The car with a speed of 120 km/h is initially located at the highway origin. With these assumptions, all trucks should only be overtaken by the car. The space–time data of all vehicles calculated by our model is given in Table 2 and depicted in Fig. 5. We summarize the findings as follows. (i) The space–time trajectory of the car crosses the trajectory of each truck only once, which means the car overtakes each truck only once. (ii) The trajectories of all trucks do not cross each other, because they are initially positioned on the highway in an ascending order of speed. (iii) At the crossing point of any two trajectories, the car's speed

Table 1
$\Delta t^{(A)}$, T and S_A given by the proposed model and the Xue–Gu model

	Vehicles overtaken					
	75 km/h	81 km/h	85 km/h	90 km/h	92 km/h	96 km/h
$\Delta t^{(A)}$(s) (proposed)	8.125	8.375	8.542	8.75	8.833	9
T(s) (proposed)	43.333	51.538	58.573	70	75.714	90
S_A(m) (proposed)	1011.4	1271.5	1497	1866.9	2052.9	2520
$\Delta t^{(A)}$(s) (Xue–Gu)	11.25	9.75	8.76	7.5	6.99	6
T(s) (Xue–Gu)	60	60	60	60	60	60
S_A(m) (Xue–Gu)	1625	1675	1708.3	1750	1766.7	1800

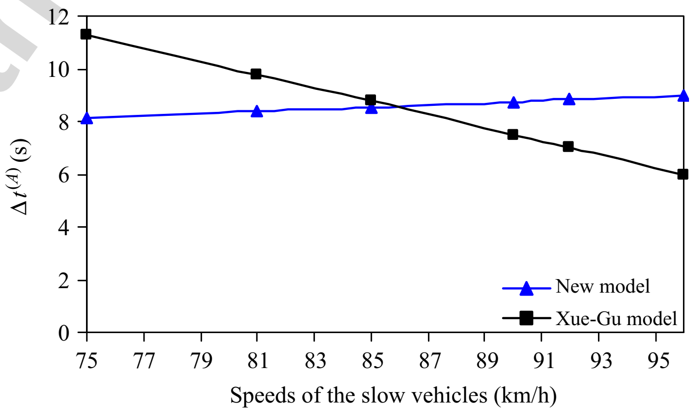

Fig. 2. Time losses of the car in overtaking the slow vehicles.

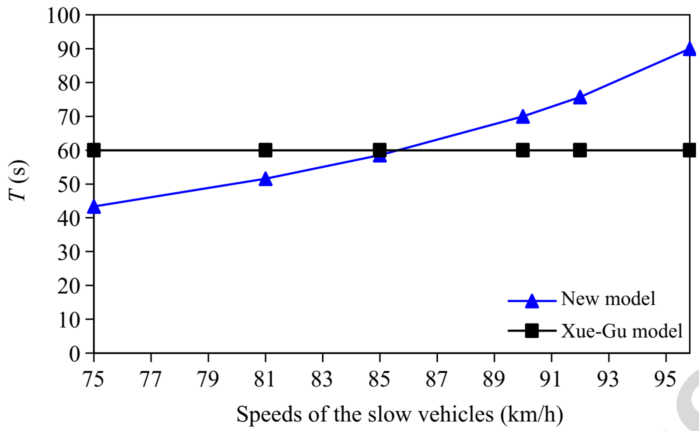

Fig. 3. The times required for overtaking the slow vehicles.

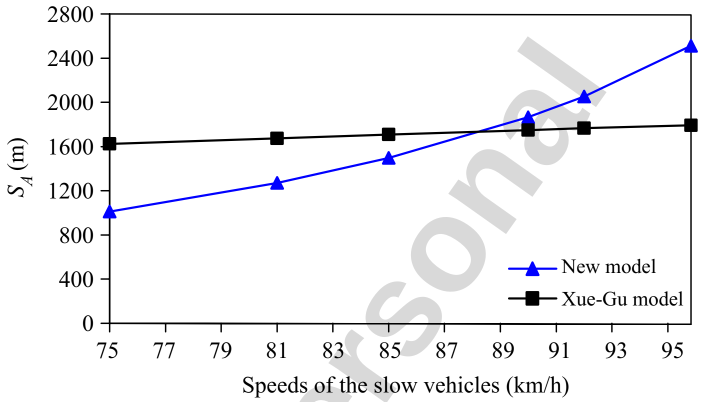

Fig. 4. Overtaking distance given by the two models.

Table 2
Space–time data of all vehicles

Time (s)	Positions of vehicles (m)						
	120 km/h	75 km/h	81 km/h	85 km/h	90 km/h	92 km/h	96 km/h
0	0	250	500	750	1000	1250	1500
50	1446	1292	1625	1931	2250	2528	2833
100	2873	2333	2750	3111	3500	3806	4167
150	4338	3375	3875	4292	4750	5083	5500
200	5833	4417	5000	5772	6000	6361	6833
250	7341	5458	6125	6653	7250	7639	8167
300	8863	6500	7250	7833	8500	8917	9500
350	10334	7542	8375	9014	9750	10194	10833
400	11914	8583	9500	10194	11000	11472	12167
450	13447	9625	10625	11375	12250	12750	13500
500	14946	10667	11750	12556	13500	14028	14833

equals the truck's speed. (iv) Nearby each of these crossing points, the car's deceleration, overtaking and acceleration are clearly reproduced.

The above example allows the slow vehicles to be overtaken only once by the car. We now investigate the case where a vehicle may be overtaken by other two or more vehicles. For simplicity, we assume that there are

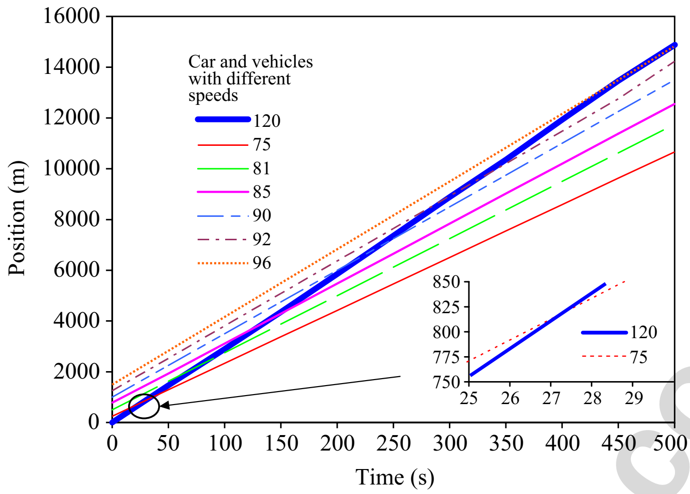

Fig. 5. Space–time trajectories of all vehicles.

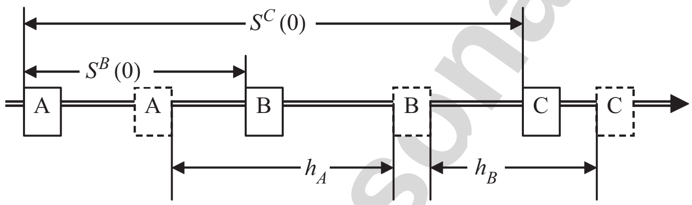

Fig. 6. Movements of vehicles A, B and C.

three vehicles A, B and C on the highway, with the maximum speeds $v_{\max}^{(A)}, v_{\max}^{(B)}$ and $v_{\max}^{(C)}$, respectively, and $v_{\max}^{(A)} > v_{\max}^{(B)} > v_{\max}^{(C)}$ holds. Their initial positions on the highway are $S^A(0), S^B(0)$ and $S^C(0)$, respectively, and $S^A(0) < S^B(0) < S^C(0)$. With these settings, the following two situations may occur: (a) vehicle B overtakes C, then A overtakes C and finally A overtakes B; (b) vehicle A overtakes B, then A overtakes C and finally B overtakes C.

In situation (a), there exist four vehicle orders on the highway, i.e., ABC, ACB, CAB and CBA. In situation (b), the four orders are ABC, BAC, BCA and CBA. Hence, we have to determine which order will first occur, ACB or BAC? Fig. 6 depicts the initial positions of the three vehicles A, B and C, where $S^A(0) = 0$. The time required by A for approaching the back of B at a safe distance of h_A, can be computed by

$$v_{\max}^{(A)} t + h_A = v_{\max}^{(B)} t + S^B(0), \tag{12}$$

which gives

$$t_{A|B} = \frac{S^B(0) - h_A}{v_{\max}^{(A)} - v_{\max}^{(B)}}. \tag{13}$$

Similarly, the time required by B for approaching the back of C at a safe distance of h_B, is

$$t_{B|C} = \frac{S^C(0) - S^B(0) - h_B}{v_{\max}^{(B)} - v_{\max}^{(C)}}. \tag{14}$$

By comparing $t_{A|B}$ and $t_{B|C}$, we can confirm which overtaking maneuver will first occur. Note that when $t_{A|B} = t_{B|C}$, vehicle B first overtakes C, before this overtaking, however, vehicle A must decelerate for a short while for keeping the safe distances apart from B and C.

Table 3
Space–time data of vehicles A, B and C

Time (min)	Positions of vehicles (m)		
	A	B	C
0	0	4000	6000
1	2000	5600	7250
2	4000	7200	8500
3	6000	8800	9750
4	8000	10400	11000
5	10000	12000	12250
6	12000	13477.3	13500
7	14000	14839.6	14750
8	15947	16439.6	16000
9	17566.8	18039.6	17250
10	19566.8	19639.6	18500
11	21251.6	21239.6	19750
12	23086.6	22839.6	21000
13	25086.6	24439.6	22500

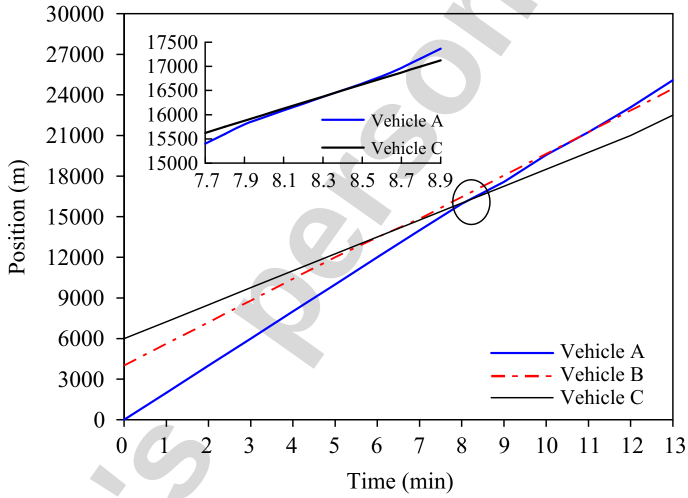

Fig. 7. Space–time trajectories of vehicles A, B and C.

Let the highway length be 25 km, and $V_{\max}^{(A)} = 120$ km/h, $v_{\max}^{(B)} = 96$ km/h, $v_{\max}^{(C)} = 75$ km/h, $S^A(0) = 0$, $S^B(0) = 4$ km and $S^C(0) = 6$ km. From Eqs. (13) and (14), we have $t_{A|B} = 590$ s and $t_{B|C} = 334$ s. Thus, vehicle B first overtakes C and then the situation (a) will be resulted from this initial setting. The space–time data are given in Table 3 and further depicted in Fig. 7. Three crossing points can be clearly observed in Fig. 7, where vehicle C is first overtaken by vehicle B, then by vehicle A, and finally B is overtaken by A.

4. Bidirectional overtaking on a two-lane highway

On the two-lane highway with bidirectional traffic flow, a driver who wants to overtake a vehicle in front of him/her must consider whether there is an enough space in the opposite lane. If the space is enough, the driver uses the opposite lane to implement the overtaking maneuver; otherwise, he or she has to continuously follow the preceding vehicle.

For demonstration, we here study a simple circumstance where three vehicles A, B and C are moving on a two-lane highway. Vehicles A and B, $v_{\max}^{(A)} > v_{\max}^{(B)}$, are moving on lane 2 from right to left, and vehicle C is moving on lane 1 from left to right, see Fig. 8. Suppose that vehicle A wants to overtake vehicle B through the

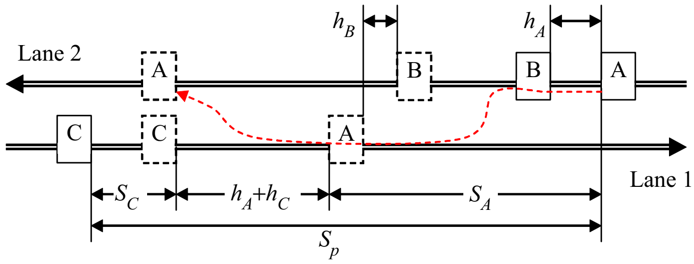

Fig. 8. Overtaking scheme in a two-lane bidirectional traffic flow.

transient use of lane 1. Clearly, for overtaking vehicle B, vehicle A should consider two safe distances h_A and h_B which correspond to the A's positions before and after having the overtaking maneuver, respectively. Let the average speed of vehicle A in overtaking be $0.5(v_{\max}^{(A)}+v_{\min}^{(A)})$. Then, the overtaking distance that vehicle A travels in overtaking becomes

$$S_A=\frac{v_{\max}^{(A)}+v_{\min}^{(A)}}{2}(T-2t_0), \tag{15}$$

where t_0 is the reactive delay time that vehicle A spends in deceleration, acceleration and lane-changing, and T is the time required for overtaking B. Using the relationship $S_A=v_{\max}^{(B)}T+h_A+h_B$, we have

$$T=\frac{2h_A+2h_B+2t_0(v_{\max}^{(A)}+v_{\min}^{(A)})}{v_{\max}^{(A)}+v_{\min}^{(A)}-2v_{\max}^{(B)}}. \tag{16}$$

Within the time duration T, vehicle C moves forward a distance S_C,

$$S_C=v_{\max}^{(C)}T. \tag{17}$$

Let S_p be the initial distance between vehicles A and C. This distance is called the passing sight distance perceived by vehicle A's driver when he/she intends to overtake B by shortly using lane 1. Obviously, for preventing the head-on collision between A and C, the following condition should be satisfied

$$S_p\geqslant S_A+S_C+h_A+h_C. \tag{18}$$

Substituting Eqs. (15)–(17) into Eq. (18), we have

$$S_p\geqslant 2h_A+h_B+h_C+\frac{2h_A+2h_B+2t_0(v_{\max}^{(A)}+v_{\min}^{(A)})}{v_{\max}^{(A)}+v_{\min}^{(A)}-2v_{\max}^{(B)}}(v_{\max}^{(B)}+v_{\max}^{(C)}). \tag{19}$$

Considering an example with such data that $v_{\max}^{(A)}=50\,\text{km/h}$, $v_{\min}^{(A)}=V_{\max}^{(B)}=30\,\text{km/h}$, $v_{\max}^{(C)}=40\,\text{km/h}$, $t_0=3\,\text{s}$, $h_A=28\,\text{m}$, $h_B=17\,\text{m}$ and $h_C=22\,\text{m}$. Using Eq. (19), we get the minimal passing sight distance of 876 m. This says, vehicle A can start overtaking vehicle B by using the opposite lane when vehicle C is apart from it more than 876 m.

5. Conclusion

We have proposed a new overtaking model that considers such important factors as the reactive delay time for vehicle acceleration, deceleration and lane-changing, the safe distance for car-following and the overtaking distance. The time required for overtaking, the time loss in overtaking procedure and the space–time evolution of vehicle movement were numerically investigated using the model and compared with the results from a survey. Numerical results show that our model can reproduce the traffic in accord with the observed one. The overtaking in a two-lane bidirectional traffic flow was also analyzed in this paper and the minimal passing sight distance for overtaking the preceding vehicle by using the opposite lane was obtained.

Acknowledgments

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